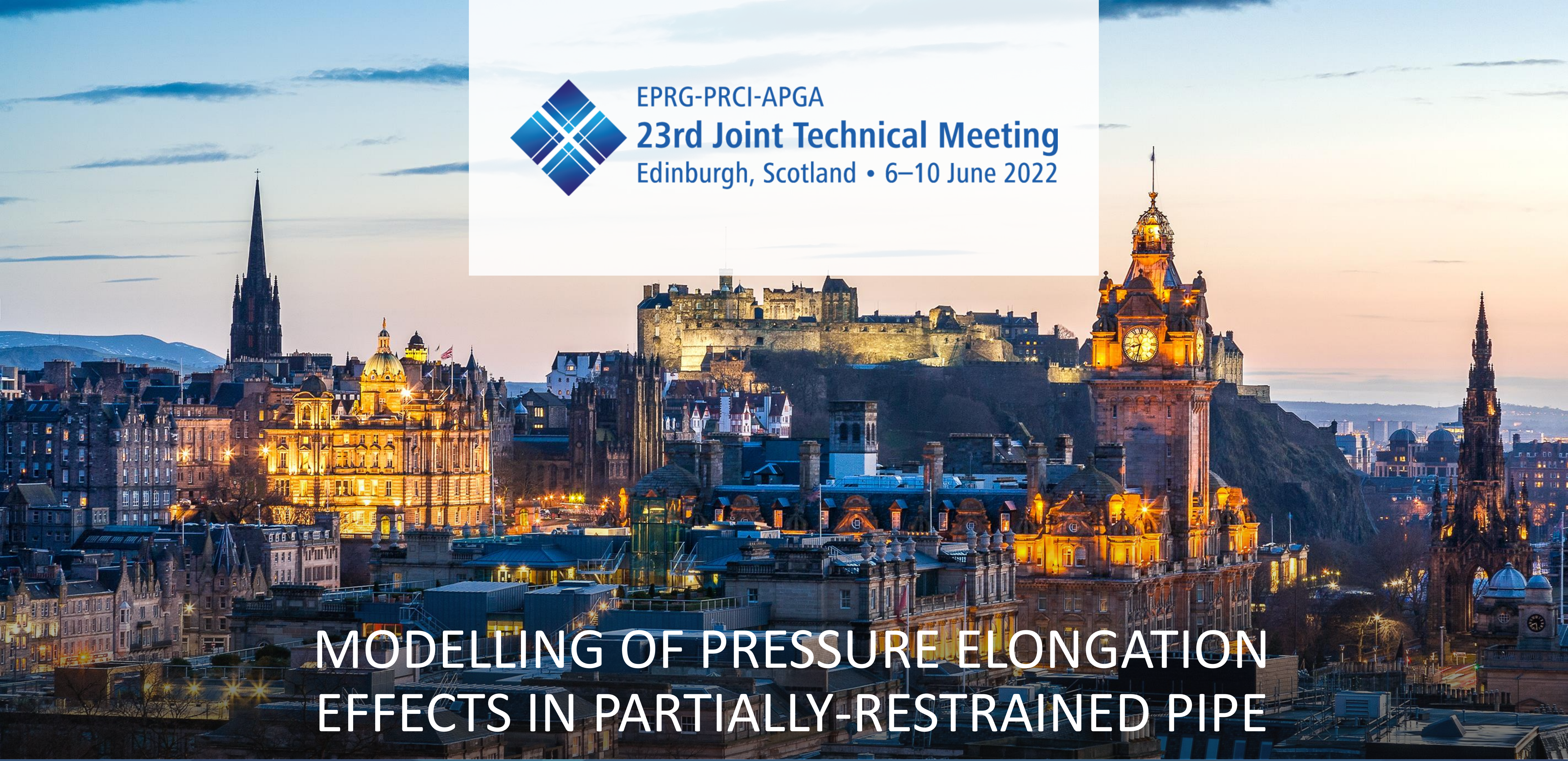
EPRG-PRCI-APGA

23rd Joint Technical Meeting

Edinburgh, Scotland • 6–10 June 2022



MODELLING OF PRESSURE ELONGATION EFFECTS IN PARTIALLY-RESTRAINED PIPE

9 JUNE 2022

It‘s about pipe stress analysis

For both *pipelines* and *piping*, **pipe stress analysis** is conducted during design,

because **stress** is *the key variable* driving many failure modes (fracture, fatigue, buckling, plastic collapse, etc.)

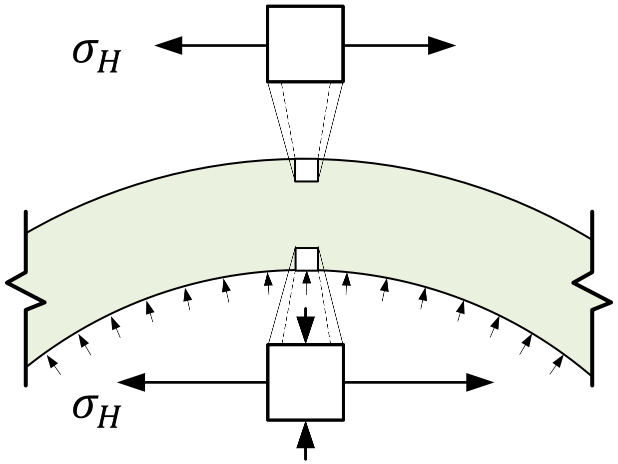
Pressure is one load considered in **stress analysis**…

Theory: stress due to pressure

Pressure causes pipe stress in three directions:

- Radial direction, σ_R – pressure is a direct compressive load inside pipe
  - Varies inner to outer surface, (from $= -P$ to $= 0$)

- Hoop direction, σ_H – pipe dilates due to pressure
  - Varies from inner to outer surface (but not much)
  - Average through thickness ($= PD_i/2t$)



$$\sigma_H = \frac{PA_i}{A_c}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_R = \frac{PA_i}{A_c}\left(1 - \frac{r_o^2}{r^2}\right)$$

NOTE: Use of $PD_o/2t$ in design factor/wall thickness equation is actually an approximation of the Tresca stress on the inner surface ($= PD_i/2t + P$).

Theory: stress due to pressure

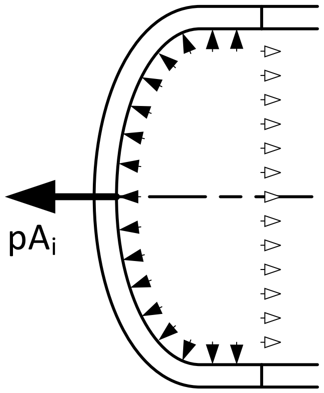
Pressure causes pipe stress in three directions:

- Two independent longitudinal effects:

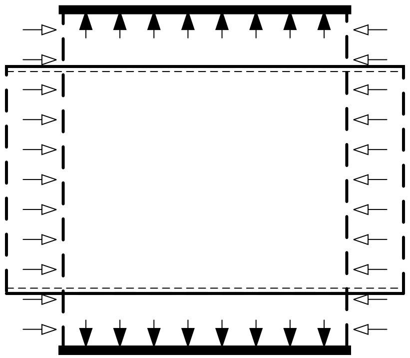
1. **End-cap *force*** – pressure applies a direct load at fittings
   - Causes stress between the applied load and reaction load
   - Automatically balanced in any closed piping system ($F = pA_i$)
   - Stress, $\sigma_L = pA_i/A_c \approx pD/4t$

2. **Poisson *strain*** – pipe seeks to *shorten* as it *dilates* due to pressure
   - Only causes stress if it is restrained, *exactly like thermal load*
   - Uniform through the thickness of the pipe. At full restraint, ($\varepsilon_L = 2\nu pA_i/A_cE$)
   - Stress, $\sigma_L = 2\nu pA_i/A_c \approx \nu\sigma_H$

End-cap



Poisson strain



Codified stress analysis methods

Pipe stress analysis is a well-established field:

- **ASME B31.3** – establishes the main method for flexible / "unrestrained" piping.
- Traditionally broad assumptions applied:
  - **Thermal** load causes only bending stress
  - **Pressure** causes only uniform tensile stress
  - Philosophy for allowable stress **permits yielding**, but **prevents plastic collapse**

Distilled to:

Design strength:

$$S = \min\left(\frac{2S_y}{3}, \frac{S_u}{3}\right)$$

Sustained:

$$S_L < S$$

Expansion:

$$\Delta S_L < 3S$$

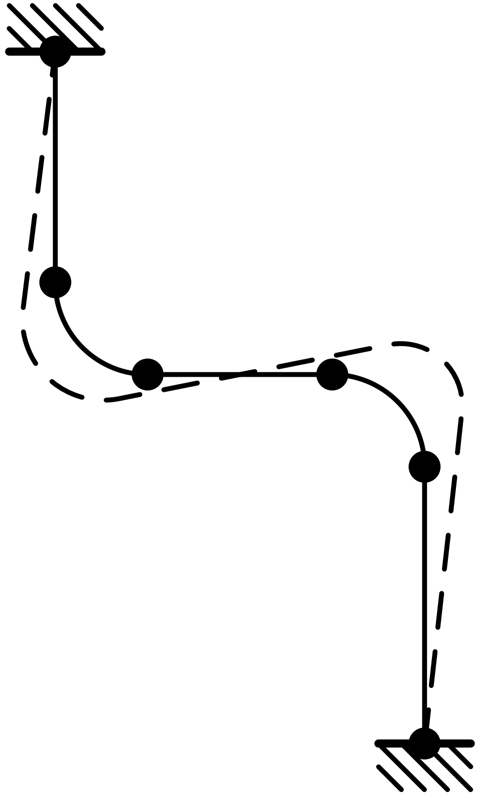
$$+ C$$

where $C = complications$

Codified stress analysis methods

Pipe stress analysis is a well-established field:

- Implementation in stress software
  - **Simulation** of thermal and weight response of a system of 1D pipe ‘elements’
  - Longitudinal pressure stress is added in **post-processing**
  - <u>Neglect any longitudinal strain/deflection due to pressure, hence no reaction loads or bending due to pressure</u>



Codified stress analysis methods

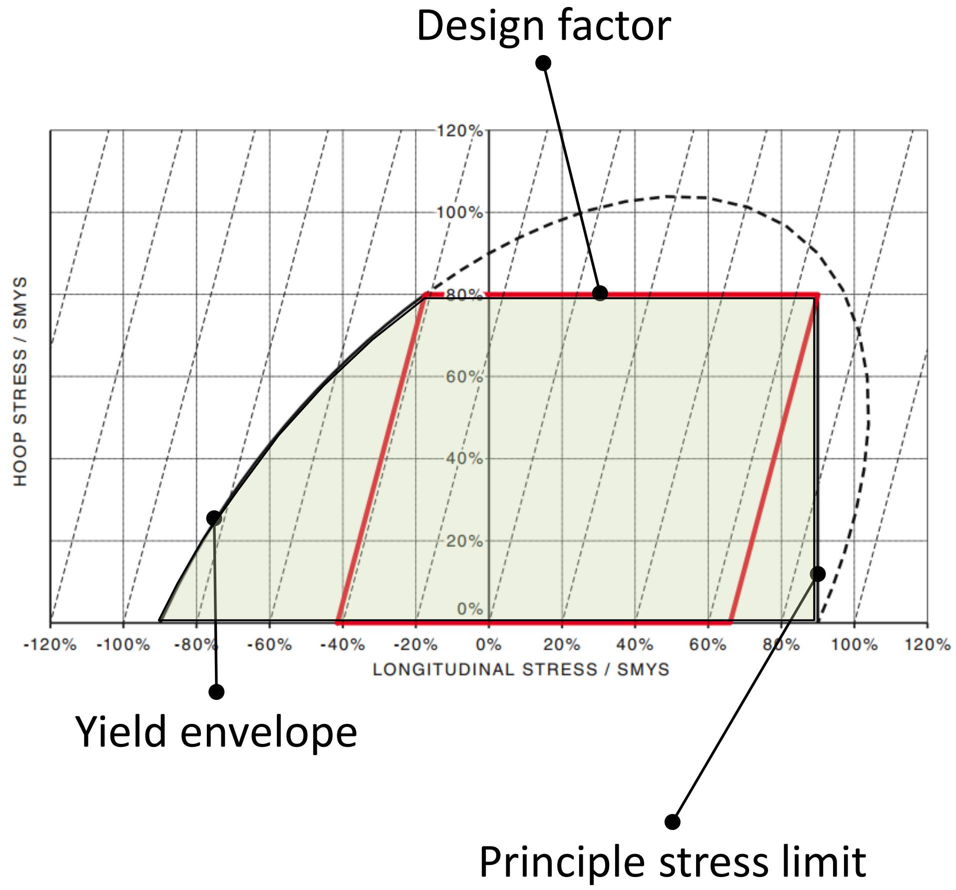
For pipelines:

- **ASME B31.4** and **8**: provide alternate method for "restrained" piping
  - Simpler analysis, applies Poisson stress term ($= \nu S_H$)

$$S_L = S_E + \nu S_H + \frac{M}{Z} + F_a A$$

  - Permits uniform load superposition, which is unexpected…

- Allowable stress
  - **Firstly:** prevention of yielding (variously $0.9Sy$)
  - **Secondly:** limits principle stress (also $0.9S_y$), suitable for weld defect loading



Codified stress analysis methods

For pipelines:

- An aside… B31.4 and B31.8 also modify the “unrestrained” allowable in two different ways…
  - 31.4 neglects ultimate strength in calculation of the allowable stress
  - 31.8 neglects yield strength in calculation of the allowable stress

ASME B31.3

$$S = \min\left(\frac{2S_y}{3}, \frac{S_u}{3}\right)$$

ASME B31.4

$$S = \frac{2S_y}{3}$$

ASME B31.8

$$S = \frac{S_u}{3}$$

Net section force method

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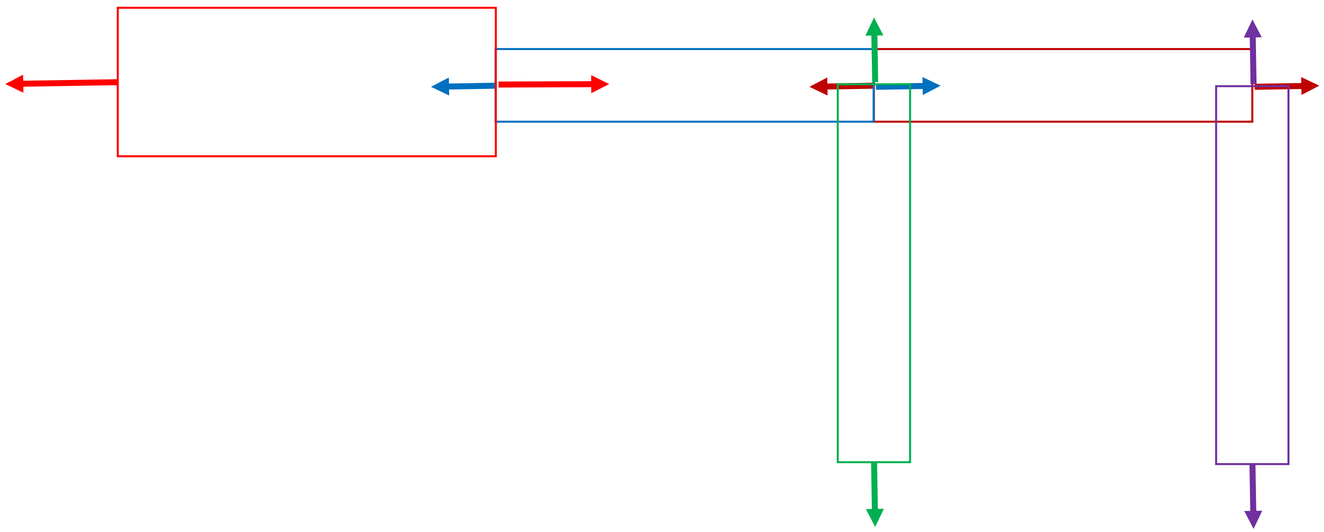
It is simple for modern software to model exact pressure elongation response:

- Poisson is a uniform strain, like temperature.
- End-cap load $= pA_i$ on every element:



Net section force method

It is simple for modern software to model exact pressure elongation response:

- The longitudinal stiffness equation is modified with both balanced end-cap and Poisson effect built-in:

Fundamental Hooke's law: $$\varepsilon_L = \frac{1}{E}(\sigma_L - \nu\sigma_L - \nu\sigma_L) + \alpha_L \Delta T$$

$$\sigma_L = \underbrace{\frac{F_a}{A_c}}_{\text{External applied forces}} + \underbrace{\frac{pA_i}{A_c}}_{\text{Internal applied force}} \qquad \nu\sigma_L + \nu\sigma_L = \underbrace{\frac{2\nu p A_i}{A_c}}_{\text{Poisson effect of pressure}}$$

Revised stiffness equation: $$\begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} = \frac{EA_c}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \left(\underbrace{\boxed{(2\nu - 1)A_i P}}_{\text{New bit}} - EA_c\alpha\Delta T\right)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Net section force method

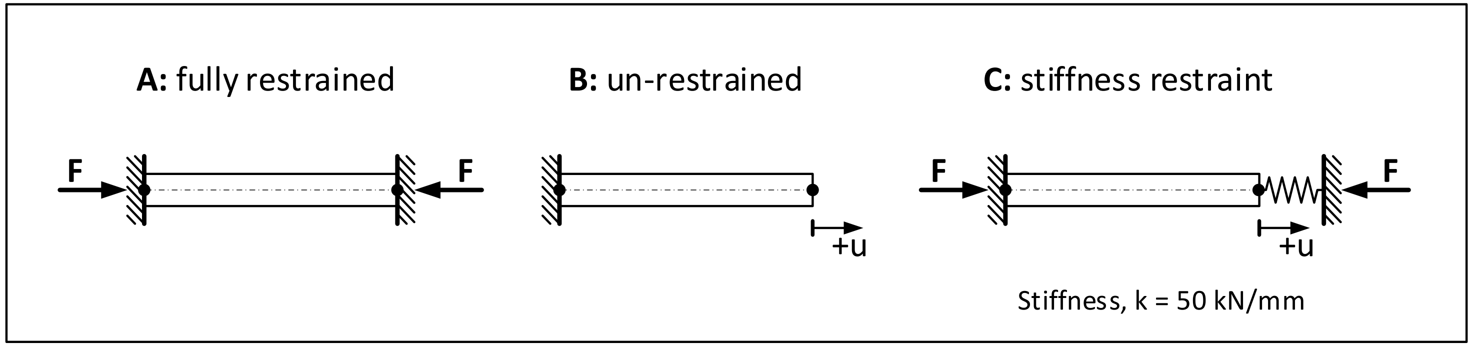
It is simple for modern software to model exact pressure elongation response:

- But, the results must be interpreted correctly.
  - **Reactions/displacements:** F_a and u will be accurate for reaction loads and.
  - **Stress** must be corrected for end-cap load:

$$\sigma_L = \frac{F_a}{A_c} + \boxed{\frac{pA_i}{A_c}}$$

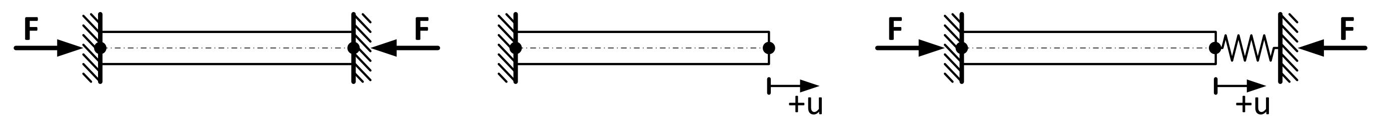
- Conceptualisation: F_a= total cross-section force *including the fluid*

Example #1



Operating pressure	P	10	MPa
Temperature change	ΔT	0	K
Outer diameter	D	219.1	mm
Wall thickness	t	7.92	mm
Modulus of elasticity	E	200	GPa
Poisson's ratio	ν	0.3	
Length	L	5	m

Example #1



Variable	**A:** fully restrained	**B:** un-restrained	**C:** stiffness restraint
Constraint	$u = 0$	$F = 0$	$F = ku$
External force, F_a	-129 kN	0 kN	-24.9 kN
Pipe force, F	195 kN	324 kN	300 kN
Pipe stress, σ_L	37.1 MPa	61.8 MPa	57.0 MPa
Displacement, u	0 mm	0.619 mm	0.499 mm

Method 2 – Zero strain datum – Paper only

For hand calculations, the following alternate method can be useful:

- Calculate the force in the pipe equivalent to zero strain:

$$F_0 = 2\nu p A_i + E\alpha\Delta T A_c$$

- Define ‘relative’ force, as the difference between the actual force and the zero-strain force:

$$F' = F - F_0$$

- Hence, all constant terms are removed from the stiffness equation:

$$F' = (A_c E)\varepsilon_L$$

Modelling of vents

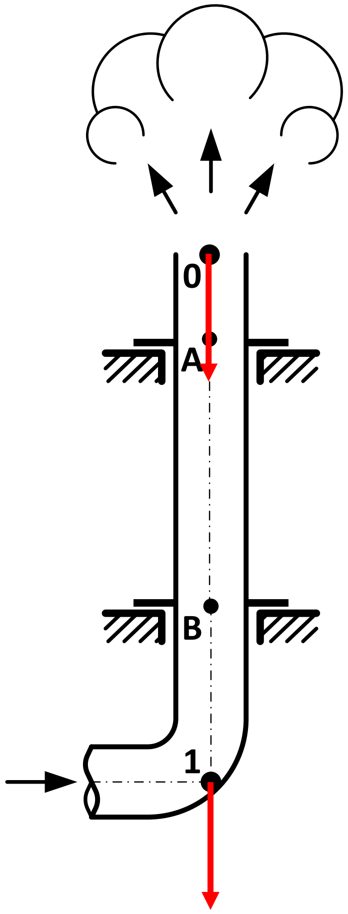
Specific issue – Vent design:

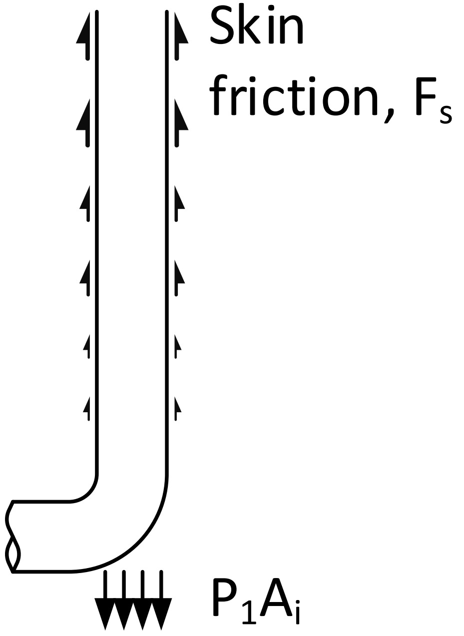
- Vents are a location where the end-cap force is *not* balanced at the next fitting, hence transfers through to… a restraint.

$$F_t = \int_{A_{c,0}} (P + \rho V^2).dA$$

$$= (\bar{P}_0 + \alpha_0 \rho_0 \bar{V}_0^2) A_i$$

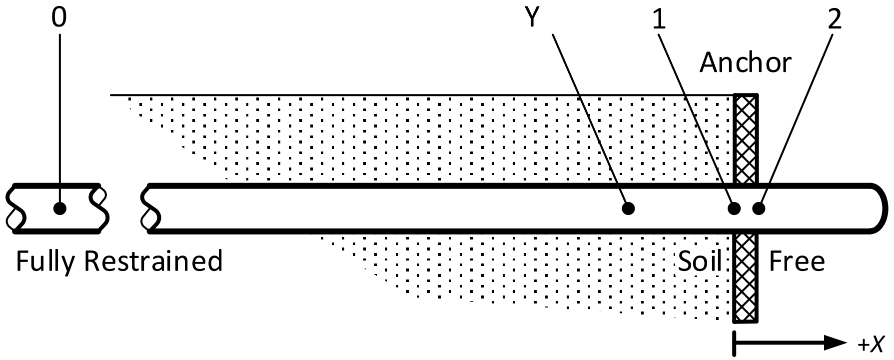
- Where should the thrust force be applied?



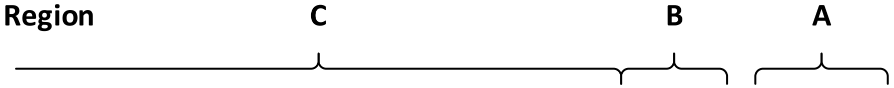


Example #2 – Paper only

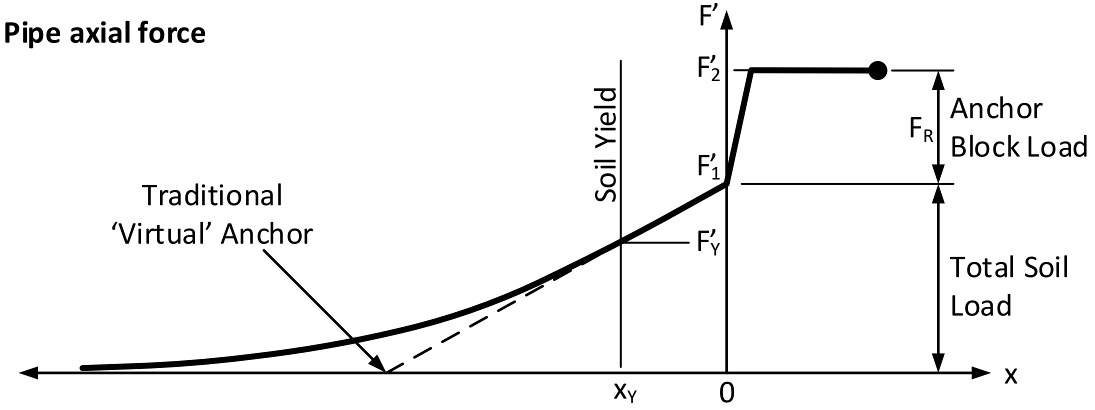
Schematic



Region



Pipe axial force



Fully-restrained datum:

From equation (10)	$F_0 = 2vpA_i + E\alpha\Delta T A_c$	
	$\boldsymbol{F_0 = 315\ \mathrm{kN}}$	

Region A: $x > 0$

Pipe force from end-cap load, eq. (3)	$F_2 = pA_i = 1{,}163\ kN$	
From equation (11)	$F_2' = pA_i - F_0$	
	$\boldsymbol{F_2' = 848\ \mathrm{kN}}$	

Region C: $-\infty < x < x_Y$

Soil reaction force for region C	$w = \frac{dF'}{dx} = ku$	(13)
From Equation (12)	$\varepsilon_L = \frac{du}{dx} = \frac{F'}{EA_c}$	
	$\therefore \int F'.dF' = \int EA_c ku.du$	
Solve, with $F' = 0$ at $u = 0$	$F'^2 = EA_c ku^2$	(14)
Hence, at the yield point	$\therefore F_Y' = \pm u_Y\sqrt{EA_c k}$	
(The positive solution is correct)	$\boldsymbol{F_Y' = 187.4\ \mathrm{kN}}$	

Region B: $x_y < x < 0$

Soil reaction force for region B	$w = \frac{dF'}{dx} = w_Y$	(15)
	$\therefore F' = \int w_Y.dx$	
Solve, with F_Y' at x_Y	$F' = F_Y' + w_Y(x - x_Y)$	(16)
Back-substitute into equation (12)	$\varepsilon_L = \frac{du}{dx} = \frac{F_Y' + w_Y(x - x_Y)}{EA_c}$	
	$\therefore u = \frac{1}{EA_c}\int (F_Y' - w_Y x_Y + w_Y x).dx$	
Solve, with u_1 at $x = 0$	$u = \left(\frac{w_Y}{2EA_c}\right)x^2 + \left(\frac{F_Y' - w_Y x_Y}{EA_c}\right)x + u_1$	(17)
Solve at x_Y	$\left(\frac{w_y}{2EA_c}\right)x_Y^2 - \left(\frac{F_Y'}{EA_c}\right)x_Y + (u_Y - u_1) = 0$	(18)
	$\boldsymbol{x_Y = -24.9\ \mathrm{m}}$	

Reaction loads

From equation (16), with $x = 0$	$F_1' = F_Y' + w_Y x_Y$	
	$\boldsymbol{F_1' = 325\ \mathrm{kN}}$	
Anchor reaction force	$F_R = F_2' - F_1'$	(19)
	$\boldsymbol{F_R = 524\ \mathrm{kN}}$	

Why pressure elongation (can) matter

Two problems:

1. Calculation of pressure elongation effects is possible with software, but **not permitted by prescriptive codes.**
   - Some software *overrides* more accurate solutions, for code compliance.
   - Incorrect interpretation may occur without guidance and a standardised method/terminology.

2. **Sometimes, partial-restraint matters**
   - End-of-line design, where there is a transition from restrained to unrestrained conditions
   - Designs using non-isotropic materials (pressure elongation can be disproportionately high for spoollable composites).
   - Modelling outputs *other* than stress. We don't only use the software for stress outputs, but also reaction loads & displacements.

Allowable stress?

Restrained pipe

- Though shalt not yield!
- Nor have a high longitudinal stress.

Unrestrained pipe

- Don't fatigue yourself, dear.
- Yield if you must,
- But make sure you don't collapse.

Allowable stress?

Partially-restrained pipe?

- As per unrestrained pipe, but
- Don’t yield, if yielding makes you worried

“where distortion might reduce a pipe’s performance, consideration shall be given to prevention of yielding by simultaneously limiting the combined stress to 0.9 x σ_y”

Conclusion

- *Permit precise methods in the standards.*
- Document methodologies, so implementation is more likely to be correct.
- Liaise with software implementers, which can be… problematic. Currently.
- For practitioners - know when these issues do and don’t matter and how to accommodate them.

Thank you for your attention.