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PAPER TITLE: MODELLING OF PRESSURE-ELONGATION EFFECTS IN PARTIALLY-RESTRAINED PIPE

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ABSTRACT

Traditional pipe stress analysis (e.g. as codified in the ASME B31 series) uses simplified rules for determining longitudinal pipe stress based on idealized fully restrained or fully unrestrained conditions. More accurate determination of longitudinal stress requires calculating the longitudinal effects of internal pressure, which include end-cap force and Poisson contraction.

Modern pipe stress analysis software permits the modelling of pressure elongation. However, without being codified, the underlying methodology and interpretation of results can lack transparency, and outputs other than stress, such as reaction loads, can become invalid.

The traditional idealized approach is acceptable for a wide range of conditions, because internal pressure does not contribute significantly to the stress state in flexible plant piping, nor restrained buried pipeline. Nevertheless, it can be significant in partially-restrained conditions. For instance, neglecting pressure can result in significant underestimate when predicting pipeline end-of-line movement in the absence of anchor blocks. It can also be significant when designs use composite materials that have anisotropic material behavior with a high hoop-to-longitudinal Poisson ratio.

This paper presents two optional conventions for modelling pressure elongation, one of which was written into the most recent version of Australian Standard AS 2885.1. A method is then applied to develop an analytical solution to predict end-of-line movement and anchor reaction forces for pipelines, using both isotropic and an orthotropic material models.

DISCLAIMER

1. BACKGROUND – TRADITIONAL PIPE STRESS ANALYSIS

Pipe stress analysis is a well-established field of engineering, being required at some level in almost every industry, from power generation to food production. The methods used for analysis have a rich legacy. In the United States, the main methodology was developed in the B31 codes of the American Society for Mechanical Engineering (ASME), and most notably ASME B31.3 *Pressure piping* [1]. In Australia, these ASME codes have also enjoyed widespread use, though because the Australian Standards have a more direct legacy from British Standards, the Australian pressure piping code, AS 4041 [2] drew on British Standard 806 for design allowable stress of steel piping. The Australian petroleum pipeline standard, AS 2885.1 [3], however, generally replicates the American liquid pipeline standard ASME B31.4 [4] for stress analysis of pipelines, and permits either ASME B31.3 or AS 4041 for facility piping.

Before the advent of modern computing, pipe stress analysis was simplified according to several paradigms, which were necessary for reliable and safe implementation. Most significantly, a broad distinction was made between restrained and unrestrained piping; ASME B31.4 is an example of a piping code that uses this categorization.

1.1. Restrained piping

"Restrained" piping is restrained so that there is zero longitudinal strain. This would be suitable *inter alia* for piping that is fixed between rigid anchors, or long runs of straight piping (e.g. most of a pipeline). In restrained piping, any load that would usually cause longitudinal strain—which includes thermal expansion and the Poisson effect—must be countered by reaction force from an anchor block or the soil or whatever is causing the restraint. Consequently, the longitudinal stress equations are a simple re-arrangement of Hooke's law, with longitudinal strain set to zero:

$$\varepsilon_L = \frac{1}{E}(\sigma_L - \nu\sigma_R - \nu\sigma_H) + \alpha\Delta T = 0$$

$$\therefore \ \sigma_L = \nu(\sigma_R + \sigma_H) + E\alpha\Delta T \qquad (1)$$

ASME B31.4 applies this solution in clause 402.6.1, though the radial stress is neglected and the Barlow formula for hoop stress is based on external diameter, which both add non-real tensile stress. ASME B31.4 also permits superposition of applied longitudinal forces and bending moments to the solution. Strictly speaking, superimposing net longitudinal forces on a pipe that is longitudinally restrained is non-physical, because the force must compromise the zero strain condition, and it is difficult to conceive of scenarios where this term ought to be used.

In this paper, a more precise derivation of longitudinal pressure stress is applied, based on Lamé's equations (refer Appendix F of AS 2885.1). This expression is simple and the resultant stress is uniform throughout the pipe cross-section.

$$\nu(\sigma_R + \sigma_H) = 2\nu P \frac{A_i}{A_c} \qquad (2)$$

1.2. Unrestrained piping

"Unrestrained" piping is an idealized concept of flexible piping arrangements used in the majority of above-ground facilities and factories. A simplified paradigm was historically applied, which assumed:

- Thermal expansion only causes lateral bending of the pipe
- Internal pressure only causes longitudinal tension in the pipe

These both warrant further discussion. Thermal expansion applies a uniform strain on piping; if it results in stress in the pipe, such stress is actually a reaction caused by restraints that prevent the pipe growing or contracting. Due to geometry, pipe is stiffer in longitudinal tension/compression than in bending, so in a flexible piping arrangement, the low bending stiffness is used intentionally to provide flexibility that relieves thermal expansion stress. The objective of design analysis is to ensure it can do so without unacceptably high stress that may damage the pipe.

In contrast to thermal expansion, Pressure directly applies a force on pipe at every change in direction. Pressure forces are inherently balanced in any closed geometry, hence any net pressure force on a piping system only results where there are open ends (e.g. thrust force at vents).

If each pipe fitting (e.g. a bend or tee) is considered in isolation, however, each opening in the fitting leaves some pressure load unbalanced on the other side. The magnitude of the unbalanced force due to each connection onto the fitting will be the internal pressure multiplied by the internal cross-sectional area of the opening. This is commonly called "end-cap force", named for the simplest fitting used to demonstrate it. If this force is transferred to the pipe, it will cause a longitudinal stress approximately half the hoop stress in magnitude.

$$F_L = PA_i$$
$$\therefore \sigma_L = P\frac{A_i}{A_c} \qquad (3)$$

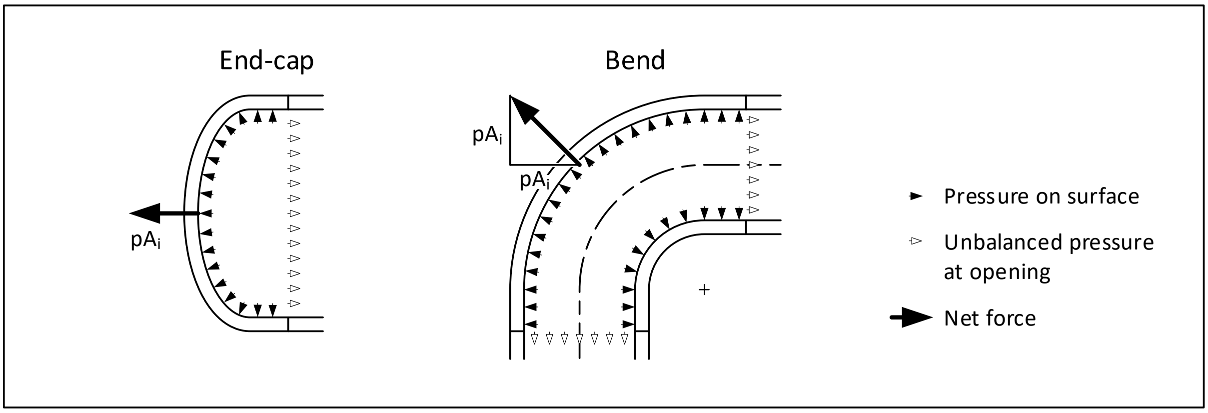

Figure 1 "End-cap" force is due to unbalanced pressure force at fittings

In reality, this longitudinal stress from pressure will cause the pipe to also grow longitudinally, which would cause reaction forces at restraints, and hence would also cause bending. Yet displacement caused by pressure was traditionally neglected. Firstly, because around 60% of the strain is counteracted by the Poisson effect of pressure (for materials with Poisson ratio of approximately 0.3), and secondly because the magnitude of the resulting stress is relatively low due to low design factors and low-strength pipe being more common in relevant applications.

Hence, the two traditional assumptions made for "unrestrained" piping were reasonable approximations for analysis for most systems, and especially for high-temperature-range systems (e.g. steam piping in power stations, or cryogenic systems) which are among the most challenging piping systems to design.

1.3. Stress analysis software

Designers of modern piping systems almost exclusively use dedicated software for piping stress analysis. The basic analysis software will undertake a simple linear finite-element analysis of 1D piping elements. Then, if standardized designs are not being applied, more though analysis of local stresses on fittings and restraints is sometimes undertaken using FEA with 2D or 3D elements.

With modern computing capabilities, the assumptions made in the above "unrestrained" paradigm can be overridden. Additionally, with the use of more detailed modelling, the realistic restraint condition can be modelled, eliminating the need for a bifurcated restrained/unrestrained categorization.

Three problems emerge with a software implementation to achieve stress compliance:

1. The first is that many piping codes do not expressly allow a more accurate determination of stress, so computing capabilities may remain underutilized for the sake of code compliance. For instance, it is most common to model the thermal expansion loads using 1D FEA, but then to superimpose the pressure stress as a blanket term based on Equation (3) above (or the more common simplified form, $pD/4t$), rather than modelling the pressure elongation. If pressure *is* modelled, some software implementations will even *override* the more accurate solution to achieve strict code compliance.

2. The second issue that emerges is that software is not only used to assess stress, but to determine both displacement and reaction forces, which are used as part of the larger facility civil and structural design. However, if the underlying analysis does not include pressure elongation effects, then the reaction forces and displacements will be incorrect. Unfortunately, reaction forces can remain incorrect or even become more incorrect when pressure elongation *is* implemented, because they might not be inferred so simply from the calculation outputs as for a typical finite element analysis.

3. As the bifurcated distinction between "restrained" and "unrestrained" is no longer necessary, a third issue emerges, that it becomes unclear which allowable stress paradigm ought to be applied in each context.

One solution to these issues is to embrace more detailed modelling potential in the piping design standards themselves, by codifying the methodology that is being applied and hence providing a reference point for the way the results should be assessed and described. Progress in this direction was made in the 2018 revision of AS 2885.1, in which "partially-restrained" piping was included as a category (Clause 5.7.2.4(c)), and an accurate pipe stiffness equation that captures pressure elongation was included in the Appendix (Equations F4.4.3(1), (2) and (3)).

2. MODELLING PRESSURE ELONGATION

2.1. Method 1 – total section force

Accurately modelling the pressure elongation in a 1D FEA requires that the end-cap force and Poisson effect are both accounted for in the pipe stiffness equation.

Although end-cap force is, in reality, applied at discrete locations throughout a piping system, it can be simplified by thinking of it as applying at both ends of every pipe element. If two elements are aligned end to end, then the two end-cap forces at the joint will cancel each other out. It becomes apparent that at any fitting, the resultant force will be a correct representation of the unbalanced force at that fitting, which is shown in Figure 2.

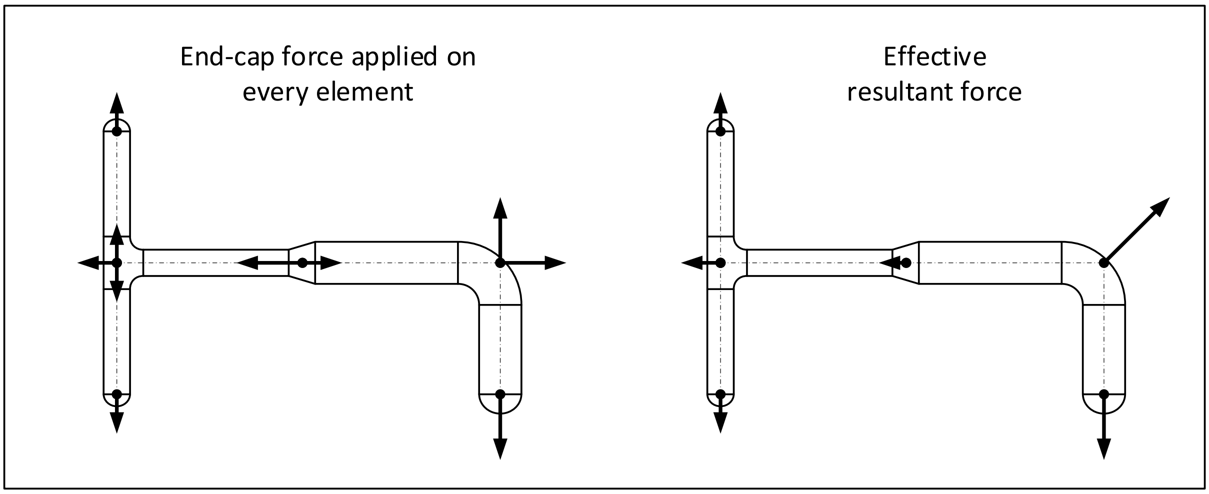

Figure 2 End-cap force applied on every pipe element will create the correct resultant force at each node for the corresponding fitting.

The stiffness equation for the pipe can be derived from Hooke's law incorporating each load term:

$$\varepsilon_L = \frac{1}{E}(\sigma_L - \nu\sigma_H - \nu\sigma_R) + \alpha\Delta T$$

Externally applied load: $\sigma_{L,1} = F_a/A_c$

End-cap force: $\sigma_{L,2} = PA_i/A_c$

Poisson effect: $\nu\sigma_H + \nu\sigma_R = 2\nu PA_i/A_c$

Hence,

$$\varepsilon_L = \frac{1}{E}\left[\frac{F_a}{A_c} + (1-2\nu)P\frac{A_i}{A_c}\right] + \alpha\Delta T \qquad (4)$$

The applied force term, F_a, contributes any external forces being applied to the pipe and should be used for reaction loads. Similarly the longitudinal strain, ε_L, is a true term representing the strain in the pipe and predicts the elongation and displacement.

A representation of this suitable for the local longitudinal force in a finite element of length L between two nodes (node '1' and node '2'), is provided in Equation (5):

$$\begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} = \frac{EA_c}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \left((2\nu-1)A_iP - EA_c\alpha\Delta T\right)\begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad (5)$$

As can be seen, the end-cap and Poisson effects, though they are very different, have combined to form a constant term similar to the thermal expansion term. When the results are interpreted for the pipe material, however, the difference between these effects is important. The stress and force in the pipe material must be calculated taking into account the end-cap force.

$$F = F_a + pA_i$$

$$\sigma_L = \frac{F_a}{A_c} + P\frac{A_i}{A_c} \qquad (6)$$

One way to conceptualize this method is that F_a is the total force across the pipe cross-section, including both the longitudinal stress in the pipe wall and the compressive force in the fluid (from fluid pressure, equal to $-pA_i$).

2.1.1. Example – one pipe element

A single pipe element is examined to demonstrate the use of the modified stiffness equation. The inputs for this example are as follows:

Operating pressure	P	10	MPa
Temperature change	ΔT	0	K
Outer diameter	D	219.1	mm
Wall thickness	t	7.92	mm
Modulus of elasticity................................	E	200	GPa
Poisson's ratio	ν	0.3	
Length..	L	5	m

The stiffness equation for this element will be according with Equation (7), applying S.I. units.

$$F_a = \frac{EA_c}{L}u + \left((2\nu - 1)A_iP - EA_c\alpha\Delta T\right)$$

$$F_a = (2.1 \times 10^8)u - (1.3 \times 10^5) \quad (7)$$

Three different restraint scenarios are assessed for the pipe element. The scenarios are illustrated in Figure 3, and calculation results for each restraint scenario are reported in Table 1.

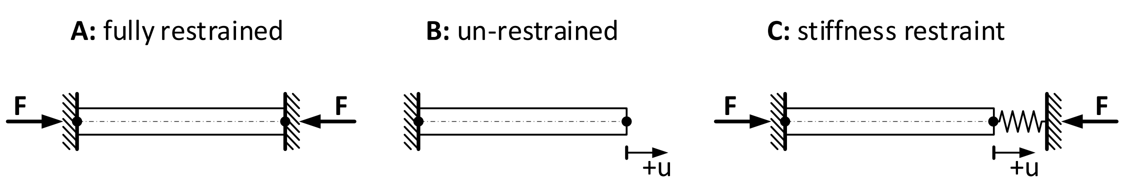

Figure 3 Three restraint scenarios analyzed.

Variable	**A:** fully restrained	**B:** un-restrained	**C:** stiffness restraint
Constraint	$u = 0$	$F = 0$	$F = ku$
External force, F_a	-129 kN	0 kN	-24.9 kN
Pipe force, F	195 kN	324 kN	300 kN
Pipe stress, σ_L	37.1 MPa	61.8 MPa	57.0 MPa
Displacement, u	0 mm	0.619 mm	0.499 mm

Table 1 Results of analysis on single pipe element in three restraint scenarios

The results demonstrate a unique feature of pressure-related pipe stress. If the pipe was unrestrained, it would expand (u in case B is positive). Consequently, the reaction force that prevents it expanding is compressive (F_a in case A is negative). Yet the stress in the pipe itself is tensile (σ_L in case A is positive). This leads to the counter-intuitive possibility that a restrained pipe could buckle due to pressure, despite the material being in a state of tension. This unusual outcome has been explored in several papers, particularly in the context of off-shore pipeline installation, where it becomes more relevant [5] [6].

2.1.2. Open pipe ends and thrust force location

Open pipe ends, such as vents, pose a complexity for pipe stress analysis. The thrust force due to the open pipe end must be balanced by reaction loads from piping restraints. The net thrust force at a vent will be the sum of the momentum flow-rate leaving the system and the net pressure force at the opening.

$$F_t = \int_{A_{c,0}} (P + \rho V^2).dA = (\bar{P}_0 + \alpha_0 \rho_0 \bar{V}_0^2) A_i \quad (8)$$

The question arises where this thrust load ought to be applied on the system. The question has only a little effect on pipe stress, but in some scenarios it would impact which restraint takes the load – for instance restraint node 'A' or node 'B' in Figure 4.

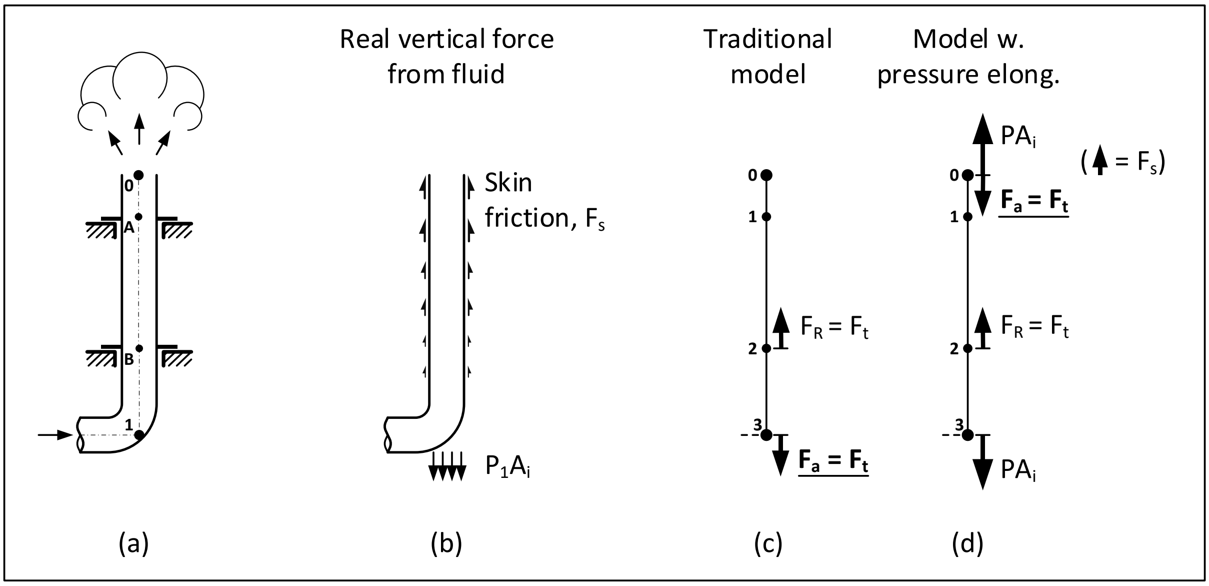

Figure 4 Modelling thrust force at open pipe ends – (a) schematic of vent, (b) real vertical force balance, (c) traditional FEA representation with no pressure elongation model and thrust applied at node 1, (d) FEA representation with pressure elongation and thrust applied at node 0.

In reality, most of the thrust load applies at the next fitting as 'end-cap load' which is not balanced at the open end, though some of it is relieved by skin friction along the pipe, which applies a small tension force in the pipe, as is illustrated in Figure 4 (b). From balance of forces, the skin friction and end-cap load must add up to the thrust force:

$$\Sigma F_L: \quad F_s = P_1 A_i - F_t \quad (9)$$

When the traditional stress analysis method is applied and pressure effects are not modelled, the next fitting is the best location to apply the thrust. If it were applied at the pipe opening, the pipe would be in compression in the model. By applying it at the next fitting, the pipe is in tension and the reaction force will be provided by the most physically representative restraint. This is shown in Figure 4 (c).

Alternatively, if pressure elongation effects have been included in the model using the method outlined in this paper, then it is more representative to apply the thrust load at the opening, node '0', where it would negate some of the end-cap load applied at that node. This is shown in Figure 4 (d). Effectively, this creates a simplified representation in which all of the skin friction applies at the end of the pipe rather than distributed along it.

Unfortunately, because modelling still neglects the fact that the pressure actually decreases gradually along the pipe when venting, it remains possible that reaction loads could be wrongly distributed in some circumstances. For example, if restraints 'A' and 'B' were both anchors then the Poisson effect under full pipe pressure modelled on the element between them would be overestimated and would hence redistribute load from one to the other more than would occur in reality. It is important for modelers to be aware of the underlying assumptions in the software implementation they are using. However, in most circumstances, vent supports are simple and the pipe should not be over-constrained. That is, good design practice should avoid ambiguity in load distribution.

2.2. Method 2 – zero strain datum

A second method to account for pressure elongation effects that may be especially convenient for analytical calculations is to consider the idealized fully-restrained pipe condition as a datum for the longitudinal force in the pipe. In this method, compared to the previous, the end-cap force is not applied automatically but must be applied manually.

The condition in the pipe material resulting in zero-strain is denoted with subscript '0' and is derived from Equations (1) and (2).

$$\sigma_{L,0} = 2\nu P \frac{A_i}{A_c} + E\alpha\Delta T$$

$$\therefore F_0 = \sigma_{L,0} A_c = 2\nu P A_i + E\alpha\Delta T A_c \quad (10)$$

A 'relative force' is then defined, as the difference between the actual longitudinal force in the pipe wall, and the zero-strain datum.

$$F' = F - F_0 \quad (11)$$

This allows Hooke's law to be simplified to the following stiffness equation, which has no constant term.

$$F' = (EA_c)\varepsilon_L \quad (12)$$

2.2.1. Example – Anchor block design

The use of this second method is here demonstrated for an analytical assessment of a pipeline end-of-line design. For simplicity it is assumed that the pipeline is straight and is fully unrestrained at one end, as shown in Figure 5. An anchor block is installed at the end of the pipeline, and the pipeline anchor block is modelled as having a finite displacement load capacity, meaning that the anchor will also displace longitudinally. The soil reaction load on the pipeline is represented by a bi-linear soil spring model, as per the PRCI seismic design guidelines [7]; refer to Figure 6.

The inputs of this calculation are summarized as follows:

Operating pressure	P	15.3	MPa
Temperature change	ΔT	25	K
Outer diameter	D	323.9	mm
Wall thickness	t	6.4	mm
Modulus of elasticity.................................	E	200	GPa
Poisson's ratio	ν	0.3	
Coefficient of thermal expansion.............	a	12	10^{-6}/K
Maximum soil resistance	w_Y	5.5	kN/m
Soil yield displacement	u_Y	5	mm
Permissible anchor movement	u_1	10	mm

The pipe is divided up into three regions with different restrain conditions. The fully-restrained condition, '0', is an asymptotic 'far field' condition as $x \to -\infty$. At some location, designated $x = x_Y$, the soil reaction reaches yield condition and hence to the left of this location the soil reaction force is linear function of displacement whereas to the right it is constant. Finally, to the right of the anchor block located at $x = 0$, the pipe is unrestrained. These regions are designated A, B and C, and are illustrated in Figure 5.

Refer to the next two pages for the analytical solution of this problem, predicting an anchor load of 524 kN at 10mm anchor displacement. This type of calculation output can be used in Civil engineering design of anchor blocks, with suitable conservatism. Additionally, a similar calculation can be used to predict the displacement of the pipeline if no anchor block is installed.

Figure 6 (c) also shows the 'virtual anchor length' that has sometimes been used in anchor block design calculations [8] [9]. The idea of a 'virtual anchor length' becomes redundant with modern computing, though it may sometimes be used to estimate what length of pipe needs to be modelled in order to achieve accurate results from an FEA (generally modelling two times the anchor length is recommended). However, good modelling practice would involve checking model convergence, rather than relying on this rule of thumb.

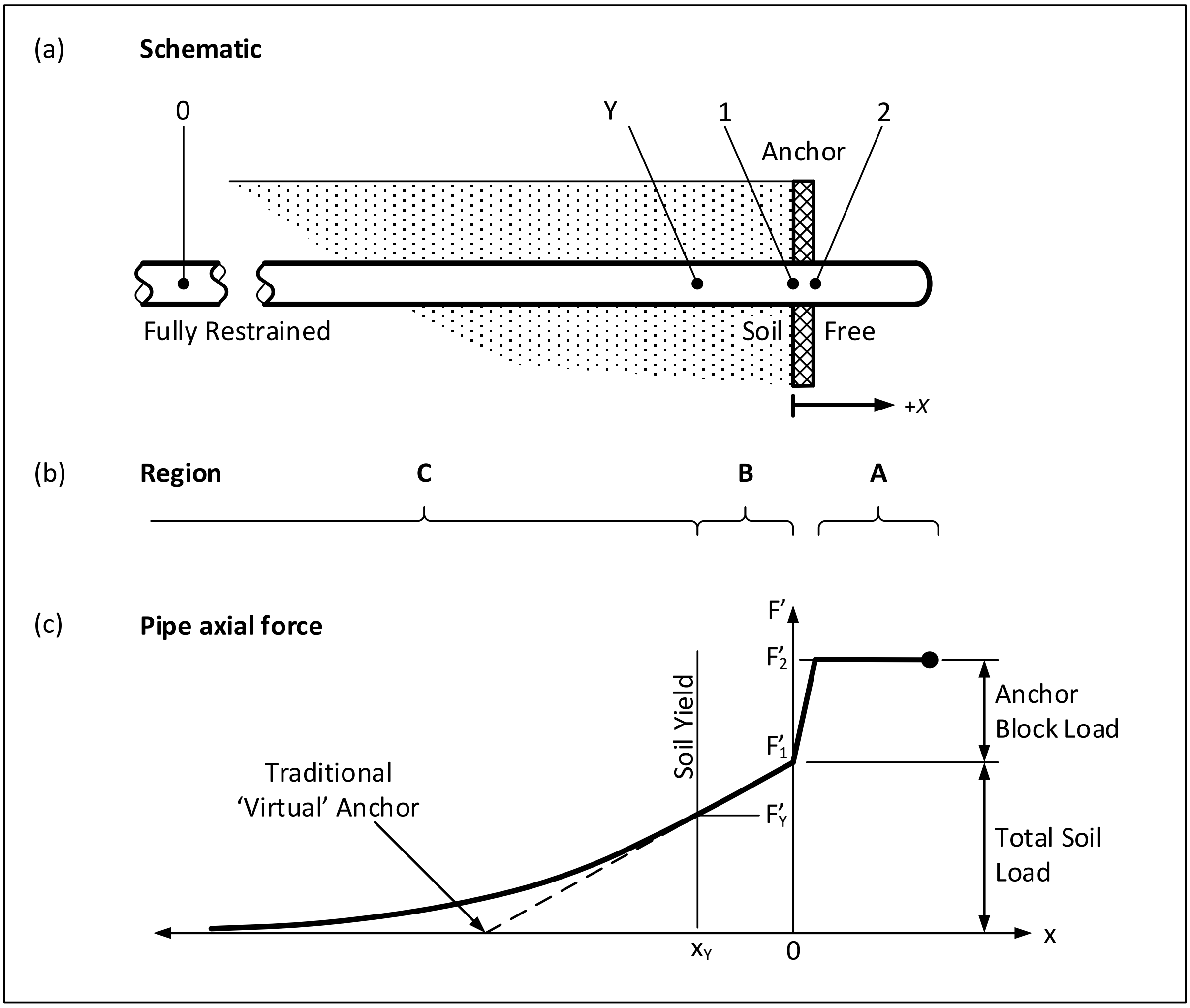

Figure 5 (a) Schematic of pipeline end-of-line example, with (b) calculation regions and (c) illustration of soil reaction force.

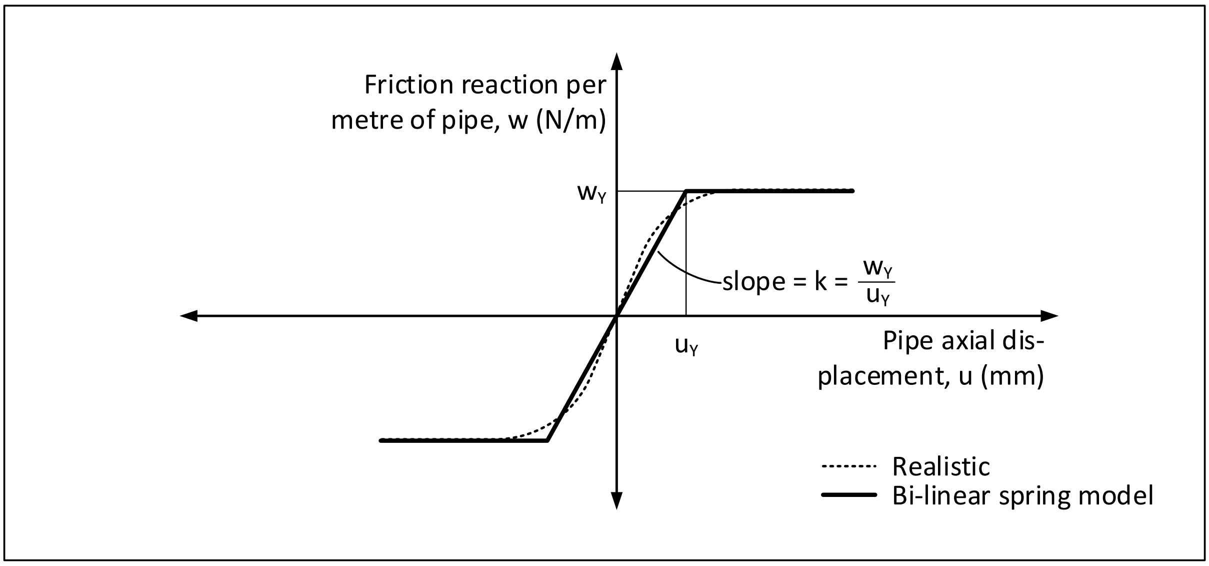

Figure 6 Soil springs approximation, consistent with PRCI Geotechnical Guidelines [7].

Fully-restrained datum:

From equation (10)	$F_0 = 2\nu pA_i + E\alpha\Delta TA_c$	
	$\boldsymbol{F_0 = 315\ \mathrm{kN}}$	

Region A: $x > 0$

Pipe force from end-cap load, eq. (3)	$F_2 = pA_i = 1{,}163\ kN$	
From equation (11)	$F'_2 = pA_i - F_0$	
	$\boldsymbol{F'_2 = 848\ \mathrm{kN}}$	

Region C: $-\infty < x < x_Y$

Soil reaction force for region C	$w = \dfrac{dF'}{dx} = ku$	(13)
From Equation (12)	$\varepsilon_L = \dfrac{du}{dx} = \dfrac{F'}{EA_c}$	
	$\therefore \int F'.dF' = \int EA_c ku.du$	
Solve, with $F' = 0$ at $u = 0$	$F'^2 = EA_c ku^2$	(14)
Hence, at the yield point	$\therefore F'_Y = \pm u_Y\sqrt{EA_c k}$	
(The positive solution is correct)	$\boldsymbol{F'_Y = 187.4\ \mathrm{kN}}$	

Region B: $x_y < x < 0$

Soil reaction force for region B	$w = \dfrac{dF'}{dx} = w_Y$	(15)
	$\therefore F' = \int w_Y.dx$	
Solve, with F'_Y at x_Y	$F' = F'_Y + w_Y(x - x_Y)$	(16)
Back-substitute into equation (12)	$\varepsilon_L = \dfrac{du}{dx} = \dfrac{F'_Y + w_Y(x - x_Y)}{EA_c}$	
	$\therefore u = \dfrac{1}{EA_c}\int (F'_Y - w_Y x_Y + w_Y x).dx$	
Solve, with u_1 at $x = 0$	$u = \left(\dfrac{w_Y}{2EA_c}\right)x^2 + \left(\dfrac{F'_Y - w_Y x_Y}{EA_c}\right)x + u_1$	(17)
Solve at x_Y	$\left(\dfrac{w_y}{2EA_c}\right)x_Y^2 - \left(\dfrac{F'_Y}{EA_c}\right)x_Y + (u_Y - u_1) = 0$	(18)
	$\boldsymbol{x_Y = -24.9\ \mathrm{m}}$	

Reaction loads

From equation (16), with $x = 0$	$F'_1 = F'_Y + w_Y x_Y$	
	$\boldsymbol{F'_1 = 325\ \mathrm{kN}}$	
Anchor reaction force	$F_R = F'_2 - F'_1$	(19)
	$\boldsymbol{F_R = 524\ \mathrm{kN}}$	

2.3. Applications

Use of pressure elongation modelling may be adopted as the routine analysis approach in lieu of the traditional idealized restraint paradigms. Those paradigms are still effective and sufficient for most scenarios, yet there are some situations in which considering pressure elongation effects become particularly important. These include:

1. Calculating reaction forces at transitions from restrained to unrestrained conditions (e.g. anchor blocks).
2. Partially-restrained pipe, such as pipeline end-of-line designs *without* anchor blocks.
3. Pipe expansion joints/bellows, which have not been addressed in this paper
4. Equipment nozzles. Equipment manufacturers or nozzle load standards may need to be consulted to determine whether allowable loads are based on inclusion or exclusion of the end-cap load.
5. Pipe in vibrating service. Design for vibrating service requires compromise between flexible and stiff design – flexible enough to limit thermal expansion stress, and stiff enough to limit vibration by increasing the natural frequency. This use of stiffer design also incurs greater pressure-related structural loads.

One condition in which pressure elongation effects can also become significant is in the design of pipelines that do not use isotropic materials, such as spirally-wound reinforced flexible polymers, and also rigid fiber-glass pipes. The most common analysis method for these pipes is to adopt an orthotropic material model, in which the longitudinal and hoop elasticity are different, and the Poisson ratio is asymmetric.

$$E_L \nu_{HL} = E_H \nu_{LH} \tag{20}$$

Added to this is the complexity of using basic geometry equivalents (A_i and A_c) in a non-homogeneous material. These materials can exhibit complex behaviors, such as contracting axially during hydrotesting (orthotropic materials can have a Poisson ratio greater than 0.5). This makes prediction of anchor loads difficult. Use of the methodologies in this paper (especially method 2) with orthotropic elastic material variables provided by the pipe manufacturer provides a way to estimate the pressure-related loads from such materials.

3. ALLOWABLE STRESS

The two broad categories, "restrained" and "unrestrained" are not only distinguished by the derivation of the stress terms, it is also usual to apply different stress limits for them.

For restrained piping, a 'no yield' criterion is commonly implemented for normal load conditions. This is achieved by calculating a representative stress for the applicable operating conditions (using either the Tresca or Von Mises formulation) and comparing this to the allowable stress, which may be as high as 90% of the Specified Minimum Yield Stress (SMYS). The principal longitudinal stress is also limited to 90% SMYS.

In contrast, loads on unrestrained piping are categorized as either sustained (resulting from applied forces) or self-relieving (resulting from applied strain). An advanced and thorough implementation of this approach is provided in ASME B31.3 *Process piping*. While sustained stress is limited based on a yield criterion, the self-relieving component is limited based on fatigue and ultimate-stress considerations. It is consequently common, especially in high-temperature systems, that the piping *will* yield the first time it reaches the most extreme service conditions, but that it will not yield again on unloading.

Of the two, the unrestrained pipe paradigm is more versatile and detailed. The restrained paradigm is intended for straight pipe and does not provide an equivalent to the stress intensity factors that were developed in parallel with the unrestrained paradigm.

ASME B31.4 includes both paradigms, but with different stress limits for unrestrained pipe than B31.3. Most notably the design factor is permitted to be as high as 0.72 (the same as unrestrained), there is no consideration of ultimate stress in calculating the allowable (only yield stress is considered), and the maximum allowable longitudinal stress is set at 75% SMYS, or 80% for occasional loads. This permits the unrestrained paradigm to be applied, yet without requiring a large step-change in pipe wall thickness, which would be problematic for a pipeline design. Interestingly, the ASME gas pipeline code, ASME B31.8 [10], differs in that it appeals to the ultimate stress in lieu of yield stress in the calculation of allowable expansion stress.

In practice, the differences between pipe that would *usually* be designed under each paradigm are:

1. Restrained pipe more often is made from line-pipe materials which have a high yield stress but also a high yield/tensile ratio.
2. Restrained pipe may more frequently use coating materials that have insufficient flexibility to tolerate the steel's yielding strain. This conclusion is anecdotal and project-specific. Importantly, however, restrained pipe is also used in a context where coating is more necessary (soil environment), and is difficult to inspect and repair.

The authors of AS 2885.1—2018 concluded that the unrestrained allowable-stress paradigm should be applied where the pipe has been classified as "partially restrained", but addressed the second item above by adding, "where distortion might reduce a pipe's performance, consideration shall be given to prevention of yielding by simultaneously limiting the combined stress to 0.9 x σ_y". If this clause were applied, it requires pipe to meet the stress limits of both paradigms at once.

When pressure elongation is modelled, though the stress derivations are changed, both these allowable stress paradigms can be applied effectively. As described above, pressure-elongation simultaneously causes end-cap load, which is a "sustained" load, and Poisson effect, which is a self-relieving strain. These two fall into different categories under the unrestrained paradigm, and Poisson effect could be assessed in the expansion stress analysis rather than the sustained stress analysis. Nevertheless, it is reasonable and practical to include both of these effects within the sustained stress assessment, which will only add some small conservatism to results (especially as sustained-case longitudinal stress is subtracted from the expansion stress allowable in the ASME formulation; refer ASME B31.3 Cl. 302.3.5, equation 1b).

4. CONCLUSION

With modern computing, accurate modelling of pressure elongation is practical, and is available from a range of existing software. It is even important for specific applications, such as pipeline end-of-line design, both with and without anchor blocks, and when calculating reaction forces and displacements in pipeline systems.

Existing piping design standards can inhibit the use of pressure elongation modelling, by imposing the use of simplified stress formula and a bifurcated restrained/unrestrained paradigm. Furthermore, silence on the issue in established pipe codes means that stress analysis software developers and piping engineers are at risk of making errors when navigating the issues that arise.

AS 2885.1 has made some progress to permit the use of accurate stress formula and pressure elongation modelling, by including a category of "partially-restrained" piping, and by providing precise pipe stress and stiffness formulas for optional use.

Further work is warranted to develop the allowable stress envelopes. Rather than providing a third "partially restrained" category, it should be possible to have one single allowable stress paradigm that is suited to all restraint conditions.

5. NOMENCLATURE

This paper adopts the following nomenclature and definitions, consistent generally with Australian Standard AS 2885.1 Appendix F. Standard international units are listed.

A_c	=	cross-sectional area of pipe material	m^2	=	$\pi(D^2 - D_i^2)/4$
A_i	=	internal area of pipe	m^2	=	$\pi D_i^2/4$
D	=	outer diameter of pipe	m		
D_i	=	inner diameter of pipe	m	=	$D - 2t$
E	=	modulus of elasticity of pipe material	Pa		
F	=	longitudinal force	N		
F_a	=	externally-applied longitudinal force	N		
F_R	=	reaction force (e.g. at anchor)	N		
F_s	=	total friction force	N		
F_t	=	total thrust force	N		
k	=	soil stiffness in linear response range (pre-yield)	N/m^2	=	w_Y/u_Y
L	=	pipe element length	m		
P	=	internal pressure in pipe	Pa		
$\bar{P}$	=	internal pressure in pipe, average across section	Pa		
t	=	pipe wall thickness	m		
T	=	pipe wall temperature	K		
u	=	longitudinal displacement	m		
u_Y	=	longitudinal displacement at soil yield	m		
V	=	fluid velocity	m/s		
$\bar{V}$	=	fluid velocity, average across section	m/s		
w	=	longitudinal soil friction per unit pipe length	N/m		
w_Y	=	longitudinal soil friction at yield point	N/m		
α	=	longitudinal coefficient of thermal expansion	K^{-1}		
α	=	velocity profile factor (thrust force equation only)			
ε_L	=	longitudinal strain	m/m ($+_{ve}$ is tensile)		
ρ	=	average fluid density	kg/m^3		
σ	=	compressive/tensile stress	Pa ($+_{ve}$ is tensile)		
σ_y	=	specified minimum yield stress (SMYS)	Pa		
σ_u	=	specified minimum tensile stress (SMTS)	Pa		
ν	=	Poisson ratio			

The following ordinates/modifies are used according to a polar cylindrical coordinate system aligned with the pipe:

L	=	pipe longitudinal direction	
x	=	pipe longitudinal coordinate	m
H	=	pipe hoop direction	
R	=	pipe radial direction	

6. REFERENCES

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